# MATHEMATICS SPECIALIST

**ATAR** COURSE

Year 12 syllabus

**Acknowledgement of Country**

Kaya. The School Curriculum and Standards Authority (the Authority) acknowledges that our offices are on Whadjuk Noongar boodjar and that we deliver our services on the country of many traditional custodians and language groups throughout Western Australia. The Authority acknowledges the traditional custodians throughout Western Australia and their continuing connection to land, waters and community. We offer our respect to Elders past and present.

# Content

# Overview of mathematics courses

There are six mathematics courses. Each course is organised into four units. Unit 1 and Unit 2 are taken in Year 11 and Unit 3 and Unit 4 in Year 12. The ATAR course examination for each of the three ATAR courses is based on Unit 3 and Unit 4 only.

The courses are differentiated, each focusing on a pathway that will meet the learning needs of a particular group of senior secondary students.

**Mathematics Preliminary** is a course which focuses on the practical application of knowledge, skills and understandings to a range of environments that will be accessed by students with special education needs. Grades are not assigned for these units. Student achievement is recorded as 'completed' or 'not completed'. This course provides the opportunity for students to prepare for post-school options of employment and further training.

**Mathematics Foundation** is a course which focuses on building the capacity, confidence and disposition to use mathematics to meet the numeracy standard for the WACE. It provides students with the knowledge, skills and understanding to solve problems across a range of contexts, including personal, community and workplace/employment. This course provides the opportunity for students to prepare for post-school options of employment and further training.

**Mathematics Essential** is a General course which focuses on using mathematics effectively, efficiently and critically to make informed decisions. It provides students with the mathematical knowledge, skills and understanding to solve problems in real contexts for a range of workplace, personal, further learning and community settings. This course provides the opportunity for students to prepare for post-school options of employment and further training.

**Mathematics Applications** is an ATAR course which focuses on the use of mathematics to solve problems in contexts that involve financial modelling, geometric and trigonometric analysis, graphical and network analysis, and growth and decay in sequences. It also provides opportunities for students to develop systematic strategies based on the statistical investigation process for answering questions that involve analysing univariate and bivariate data, including time series data.

**Mathematics Methods** is an ATAR course which focuses on the use of calculus and statistical analysis. The study of calculus provides a basis for understanding rates of change in the physical world, and includes the use of functions, their derivatives and integrals, in modelling physical processes. The study of statistics develops students' ability to describe and analyse phenomena that involve uncertainty and variation.

**Mathematics Specialist** is an ATAR course which provides opportunities, beyond those presented in the Mathematics Methods ATAR course, to develop rigorous mathematical arguments and proofs, and to use mathematical models more extensively. The Mathematics Specialist ATAR course contains topics in functions and calculus that build on and deepen the ideas presented in the Mathematics Methods ATAR course, as well as demonstrate their application in many areas. This course also extends understanding and knowledge of statistics and introduces the topics of vectors, complex numbers and matrices. The Mathematics Specialist ATAR course is the only ATAR mathematics course that should not be taken as a stand-alone course.

# Rationale

Mathematics is the study of order, relation and pattern. From its origins in counting and measuring, it has evolved in highly sophisticated and elegant ways to become the language now used to describe much of the modern world. Statistics is concerned with collecting, analysing, modelling and interpreting data in order to investigate and understand real world phenomena and solve problems in context. Together, mathematics and statistics provide a framework for thinking and a means of communication that is powerful, logical, concise and precise.

Because both mathematics and statistics are widely applicable as models of the world around us, there is ample opportunity for problem-solving throughout the Mathematics Specialist ATAR course. There is also a sound logical basis to this course, and in mastering the course, students will develop logical reasoning skills to a high level.

The Mathematics Specialist ATAR course provides opportunities, beyond those presented in the Mathematics Methods ATAR course, to develop rigorous mathematical arguments and proofs, and to use mathematical and statistical models more extensively. Topics are developed systematically and lay the foundations for future studies in quantitative subjects in a coherent and structured fashion. Students of the Mathematics Specialist ATAR course will be encouraged to appreciate the true nature of mathematics, its beauty and its functionality.

The Mathematics Specialist ATAR course has been designed to be taken in conjunction with the Mathematics Methods ATAR course. The course contains topics in functions, calculus, probability and statistics that build on and deepen the ideas presented in the Mathematics Methods ATAR course and demonstrate their application in many areas. Vectors, complex numbers and matrices are introduced. The Mathematics Specialist ATAR course is designed for students with a strong interest in mathematics, including those intending to study mathematics, statistics, all sciences and associated fields, economics or engineering at university.

For all content areas of the Mathematics Specialist ATAR course, the proficiency strands of the Year 7–10 curriculum continue to be applicable and should be inherent in students' learning of the course. These strands are Understanding, Fluency, Problem-solving and Reasoning and they are both essential and mutually reinforcing. For all content areas, practice allows students to achieve fluency of skills, such as finding the scalar product of two vectors, or finding the area of a region contained between curves, freeing up working memory for more complex aspects of problem-solving. In the Mathematics Specialist ATAR course, the formal explanation of reasoning through mathematical proof takes on an important role and the ability to present the solution of any problem in a logical and clear manner is of paramount importance. The ability to transfer skills learned to solve one class of problem, for example, integration, to solve another class of problem, such as those in biology, kinematics or statistics, is a vital part of mathematics learning in this course.

The Mathematics Specialist ATAR course is structured over four units. The topics in Unit 1 broaden students' mathematical experience and provide different scenarios for incorporating mathematical arguments and problem solving. The unit blends algebraic and geometric thinking. In this course, there is a progression of content, applications, level of sophistication and abstraction. For example, in Unit 1, vectors for two-dimensional space are introduced and then in Unit 3, vectors are studied for three-dimensional space. The Unit 3 vector topic leads to the establishment of the equations of lines and planes, and this in turn prepares students for an introduction to solving simultaneous equations in three variables. The study of calculus, which is developed in the Mathematics Methods ATAR course, is applied in vectors in Unit 3 and applications of calculus and statistics in Unit 4.

# Aims

The Mathematics Specialist ATAR course aims to develop students':

- understanding of concepts and techniques drawn from combinatorics, geometry, trigonometry, complex numbers, vectors, matrices, calculus and statistics

- ability to solve applied problems using concepts and techniques drawn from combinatorics, geometry, trigonometry, complex numbers, vectors, matrices, calculus and statistics

- capacity to choose and use technology appropriately

- reasoning in mathematical and statistical contexts and interpretation of mathematical and statistical information, including ascertaining the reasonableness of solutions to problems

- capacity to communicate in a concise and systematic manner using appropriate mathematical and statistical language

- ability to construct proofs.

# Organisation

This course is organised into a Year 11 syllabus and a Year 12 syllabus. The cognitive complexity of the syllabus content increases from Year 11 to Year 12.

## Structure of the syllabus

The Year 12 syllabus is divided into two units which are delivered as a pair. The notional time for the pair of units is 110 class contact hours.

In this course there is a progression of content, applications, level of sophistication and abstraction. For example, vectors in the plane are introduced in Year 11 Unit 1 and then in Year 12 Unit 3, they are studied for three-dimensional space. In Unit 3, the topic 'Vectors in three dimensions' leads to the establishment of the equations of lines and planes, and this in turn, prepares students for solving simultaneous equations in three variables.

## Organisation of content

### Unit 3

This unit contains the three topics:

3.1     Complex numbers

3.2     Functions and sketching graphs

3.3     Vectors in three dimensions

The Cartesian form of complex numbers was introduced in Unit 2, and in Unit 3, the study of complex numbers is extended to the polar form. The study of functions and techniques of calculus begun in the Mathematics Methods ATAR course is extended and utilised in the sketching of graphs and the solution of problems involving integration. The study of vectors begun in Unit 1, which focused on vectors in one- and two-dimensional space, is extended in Unit 3 to three-dimensional vectors, vector equations and vector calculus, with the latter building on students' knowledge of calculus from the Mathematics Methods ATAR course. Cartesian and vector equations, together with equations of planes, enables students to solve geometric problems and to solve problems involving motion in three-dimensional space.

### Unit 4

This unit contains the three topics:

4.1     Integration and applications of integration

4.2     Rates of change and differential equations

4.3     Statistical inference

In this unit, the study of differentiation and integration of functions is continued, and the techniques developed from this and previous topics in calculus are applied to the area of simple differential equations, in particular in biology and kinematics. These topics serve to demonstrate the applicability of the mathematics learnt throughout this course. Also in this unit, all of the students' previous experience in statistics is drawn together in the study of the distribution of sample means. This is a topic that demonstrates the utility and power of statistics.

Each unit includes:

- a unit description – a short description of the focus of the unit

- learning outcomes – a set of statements describing the learning expected as a result of studying the unit

- unit content – the content to be taught and learned.

### Role of technology

It is assumed that students will have access to an extensive range of technological applications and techniques. If appropriately used, these have the potential to enhance the teaching and learning of mathematics. However, students also need to continue to develop skills that do not depend on technology. The ability to be able to choose when or when not to use some form of technology and to be able to work flexibly with technology are important skills in this course.

# Representation of the general capabilities

The general capabilities encompass the knowledge, skills, behaviours and dispositions that will assist students to live and work successfully in the twenty-first century. Teachers may find opportunities to incorporate the capabilities into the teaching and learning program for the Mathematics Specialist ATAR course. The general capabilities are not assessed unless they are identified within the specified unit content.

### Literacy

Literacy skills and strategies enable students to express, interpret, and communicate complex mathematical information, ideas and processes. Mathematics provides a specific and rich context for students to develop their ability to read, write, visualise and talk about complex situations involving a range of mathematical ideas. Students can apply and further develop their literacy skills and strategies by shifting between verbal, graphic, numerical and symbolic forms of representing problems in order to formulate, understand and solve problems and communicate results. This process of translation across different systems of representation is essential for complex mathematical reasoning and expression. Students learn to communicate their findings in different ways, using multiple systems of representation and data displays to illustrate the relationships they have observed or constructed.

### Numeracy

Students who undertake this course will continue to develop their numeracy skills at a more sophisticated level, making decisions about the relevant mathematics to use, following through with calculations selecting appropriate methods and being confident of their results. This course contains topics that will equip students for the ever-increasing demands of the information age, developing the skills of critical evaluation of numerical information. Students will enhance their numerical operation skills by working with complex numbers, vectors, the calculus of functions, and by application in statistical inference problems.

### Information and communication technology capability

Students use information and communication technology (ICT) both to develop theoretical mathematical understanding and to apply mathematical knowledge to a range of problems. They use software aligned with areas of work and society with which they may be involved, such as for statistical analysis, generation of algorithms, manipulation and complex calculation. They use digital tools to make connections between mathematical theory, practice and application; for example, to use data, to address problems, and to operate systems in authentic situations.

### Critical and creative thinking

Students compare predictions with observations when evaluating a theory. They check the extent to which their theory-based predictions match observations. They assess whether, if observations and predictions don't match, it is due to a flaw in theory or method of applying the theory to make predictions – or both. They revise, or reapply their theory more skilfully, recognising the importance of self-correction in the building of useful and accurate theories and making accurate predictions.

### Personal and social capability

Students develop personal and social competence in mathematics through setting and monitoring personal and academic goals, taking initiative, building adaptability, communication, teamwork and decision making. The elements of personal and social competence relevant to mathematics mainly include the application of mathematical skills for their decision making, life-long learning, citizenship and self-management. In addition, students will work collaboratively in teams and independently as part of their mathematical explorations and investigations.

### Ethical understanding

Students develop ethical understanding in mathematics through decision making connected with ethical dilemmas that arise when engaged in mathematical calculation and the dissemination of results, and the social responsibility associated with teamwork and attribution of input. The areas relevant to Mathematics include issues associated with ethical decision making as students work collaboratively in teams and independently as part of their mathematical explorations and investigations. Acknowledging errors rather than denying findings and/or evidence involves resilience and ethical understanding. They develop increasingly advanced communication, research, and presentation skills to express viewpoints.

### Intercultural understanding

Students understand mathematics as a socially constructed body of knowledge that uses universal symbols but has its origin in many cultures. Students understand that some languages make it easier to acquire mathematical knowledge than others. Students also understand that there are many culturally diverse forms of mathematical knowledge, including diverse relationships to number and that diverse cultural spatial ability and understandings are shaped by a person's environment and language.

## Representation of the cross-curriculum priorities

The cross-curriculum priorities address contemporary issues which students face in a globalised world. Teachers may find opportunities to incorporate the priorities into the teaching and learning program for the Mathematics Specialist ATAR course. The cross-curriculum priorities are not assessed unless they are identified within the specified unit content.

### Aboriginal and Torres Strait Islander histories and cultures

Mathematics courses value the histories, cultures, traditions and languages of Aboriginal and Torres Strait Islander Peoples' past and ongoing contributions to contemporary Australian society and culture. Through the study of mathematics within relevant contexts, opportunities may allow for the development of students' understanding and appreciation of the diversity of Aboriginal and Torres Strait Islander Peoples' histories and cultures.

### Asia and Australia's engagement with Asia

There are strong social, cultural and economic reasons for Australian students to engage with the countries of Asia and with the past and ongoing contributions made by the peoples of Asia in Australia. It is through the study of Mathematics in an Asian context that students engage with Australia's place in the region. By analysing relevant data, students have opportunities to further develop an understanding of the diverse nature of Asia's environments and traditional and contemporary cultures.

### Sustainability

Each of the mathematics courses provides the opportunity for the development of informed and reasoned points of view, discussion of issues, research and problem-solving. Teachers are therefore encouraged to select contexts for discussion that are connected with sustainability. Through the analysis of data, students have the opportunity to research and discuss sustainability and learn the importance of respecting and valuing a wide range of world perspectives.

# Unit 3

## Unit description

Unit 3 of the Mathematics Specialist ATAR course contains three topics: Complex numbers, Functions and sketching graphs and Vectors in three dimensions. The study of vectors was introduced in Unit 1 with a focus on vectors in two-dimensional space. In this unit, three-dimensional vectors are studied and vector equations and vector calculus are introduced, with the latter extending students' knowledge of calculus from the Mathematics Methods ATAR course. Cartesian and vector equations, together with equations of planes, enables students to solve geometric problems and to solve problems involving motion in three-dimensional space. The Cartesian form of complex numbers was introduced in Unit 2, and the study of complex numbers is now extended to the polar form.

The study of functions and techniques of graph sketching, begun in the Mathematics Methods ATAR course, is extended and applied in sketching graphs and solving problems involving integration.

Access to technology to support the computational aspects of these topics is assumed.

## Learning outcomes

By the end of this unit, students:

- understand the concepts and techniques in vectors, complex numbers, functions and graph sketching

- apply reasoning skills and solve problems in vectors, complex numbers, functions and graph sketching

- communicate their arguments and strategies when solving problems

- construct proofs of results

- interpret mathematical information and ascertain the reasonableness of their solutions to problems.

## Unit content

An understanding of the Year 11 content is assumed knowledge for students in Year 12. It is recommended that students studying Unit 3 and Unit 4 have completed Unit 1 and Unit 2.

This unit includes the knowledge, understandings and skills described below. This is the examinable content.

### Topic 3.1: Complex numbers (18 hours)

**Cartesian forms**

3.1.1    review real and imaginary parts $\text{Re}(z)$ and $\text{Im}(z)$ of a complex number $z$

3.1.2    review Cartesian form

3.1.3    review complex arithmetic using Cartesian forms

**Complex arithmetic using polar form**

3.1.4    use the modulus $|z|$ of a complex number $z$ and the argument $\text{Arg}(z)$ of a non-zero complex number $z$ and prove basic identities involving modulus and argument

3.1.5    convert between Cartesian and polar form

3.1.6 define and use multiplication, division, and powers of complex numbers in polar form and the geometric interpretation of these

3.1.7 prove and use De Moivre's theorem for integral powers

**The complex plane (The Argand plane)**

3.1.8 examine and use addition of complex numbers as vector addition in the complex plane

3.1.9 examine and use multiplication as a linear transformation in the complex plane

3.1.10 identify subsets of the complex plane determined by relations such as

$$\left| z - 3i \right| \leq 4, \ \frac{\pi}{4} \leq Arg(z) \leq \frac{3\pi}{4} \text{ and } \left| z - 1 \right| = 2\left| z - i \right|$$

**Roots of complex numbers**

3.1.11 determine and examine the $n^{\text{th}}$ roots of unity and their location on the unit circle

3.1.12 determine and examine the $n^{\text{th}}$ roots of complex numbers and their location in the complex plane

**Factorisation of polynomials**

3.1.13 prove and apply the factor theorem and the remainder theorem for polynomials

3.1.14 consider conjugate roots for polynomials with real coefficients

3.1.15 solve simple polynomial equations

## Topic 3.2: Functions and sketching graphs (16 hours)

**Functions**

3.2.1 determine when the composition of two functions is defined

3.2.2 determine the composition of two functions

3.2.3 determine if a function is one-to-one

3.2.4 find the inverse function of a one-to-one function

3.2.5 examine the reflection property of the graphs of a function and its inverse

**Sketching graphs**

3.2.6 use and apply $\left| x \right|$ for the absolute value of the real number $x$ and the graph of $y = \left| x \right|$

3.2.7 examine the relationship between the graph of

$y = f(x)$ and the graphs of $y = \dfrac{1}{f(x)}$, $y = \left| f(x) \right|$ and $y = f(\left| x \right|)$

3.2.8 sketch the graphs of simple rational functions where the numerator and denominator are polynomials of low degree

**Topic 3.3: Vectors in three dimensions (21 hours)**

**The algebra of vectors in three dimensions**

3.3.1    define the concept of a vector in three dimensions, using the unit vectors **i, j** and **k**, determining magnitude, scalar (dot) product and parallel and perpendicular vectors

3.3.2    prove geometric results in the plane and construct simple proofs in 3 dimensions

**Vector and Cartesian equations**

3.3.3    introduce Cartesian coordinates for three dimensional space, including plotting points and equations of spheres

3.3.4    use vector equations of curves in two or three dimensions involving a parameter and determine a 'corresponding' Cartesian equation in the two-dimensional case

3.3.5    determine a vector equation of a straight line and straight line segment, given the position of two points or equivalent information, in both two and three dimensions

3.3.6    examine the position of two particles, each described as a vector function of time, and determine if their paths cross or if the particles meet

3.3.7    use the cross product to determine a vector normal to a given plane

3.3.8    determine vector and Cartesian equations of a plane

**Systems of linear equations**

3.3.9    recognise the general form of a system of linear equations in several variables, and use elementary techniques of elimination to solve a system of linear equations

3.3.10   examine the three cases for solutions of systems of equations – a unique solution, no solution, and infinitely many solutions – and the geometric interpretation of a solution of a system of equations with three variables

**Vector calculus**

3.3.11   consider position vectors as a function of time

3.3.12   derive the Cartesian equation of a path given as a vector equation in two dimensions, including ellipses and hyperbolas

3.3.13   differentiate and integrate a vector function with respect to time

3.3.14   determine equations of motion of a particle travelling in a straight line with both constant and variable acceleration

3.3.15   apply vector calculus to motion in a plane, including projectile and circular motion

# Unit 4

## Unit description

Unit 4 of the Mathematics Specialist ATAR course contains three topics: Integration and applications of integration, Rates of change and differential equations and Statistical inference.

In Unit 4, the study of differentiation and integration of functions continues, and the calculus techniques developed in this and previous topics are applied to simple differential equations, in particular in biology and kinematics. These topics demonstrate the real-world applications of the mathematics learned throughout the Mathematics Specialist ATAR course.

In this unit, all of the students' previous experience working with probability and statistics is drawn together in the study of statistical inference for the distribution of sample means and confidence intervals for sample means.

Access to technology to support the computational aspects of these topics is assumed.

## Learning outcomes

By the end of this unit, students:

- understand the concepts and techniques in applications of calculus and statistical inference

- apply reasoning skills and solve problems in applications of calculus and statistical inference

- communicate their arguments and strategies when solving problems

- construct proofs of results

- interpret mathematical and statistical information and ascertain the reasonableness of their solutions to problems.

## Unit content

An understanding of the Year 11 content is assumed knowledge for students in Year 12. It is recommended that students studying Unit 3 and Unit 4 have completed Unit 1 and Unit 2.

This unit includes the knowledge, understandings and skills described below. This is the examinable content.

### Topic 4.1: Integration and applications of integration (20 hours)

**Integration techniques**

4.1.1　integrate using the trigonometric identities

$$\sin^2 x = \frac{1}{2}(1 - \cos 2x), \ \cos^2 x = \frac{1}{2}(1 + \cos 2x) \text{ and } 1 + \tan^2 x = \sec^2 x$$

4.1.2　use substitution $u = g(x)$ to integrate expressions of the form $f\big(g(x)\big)g'(x)$

4.1.3　establish and use the formula $\int \frac{1}{x} dx = \ln|x| + c$ for $x \neq 0$

4.1.4　use partial fractions where necessary for integration in simple cases

**Applications of integral calculus**

4.1.5    calculate areas between curves defined by functions of the form $y = f(x) \; or \; x = f(y)$

4.1.6    determine volumes of solids of revolution about either axis

4.1.7    use technology to evaluate integrals numerically

## Topic 4.2: Rates of change and differential equations (20 hours)

**Applications of differentiation**

4.2.1    use implicit differentiation to determine the gradient of curves whose equations are given in implicit form

4.2.2    examine related rates as instances of the chain rule: $\dfrac{dy}{dx} = \dfrac{dy}{du} \times \dfrac{du}{dx}$

4.2.3    apply the increments formula $\delta y \approx \dfrac{dy}{dx} \delta x$ to differential equations

4.2.4    solve simple first order differential equations of the form $\dfrac{dy}{dx} = f(x)$; differential equations of the

form $\dfrac{dy}{dx} = g(y)$; and, in general, differential equations of the form $\dfrac{dy}{dx} = f(x)g(y)$, using

separation of variables

4.2.5    examine slope (direction or gradient) fields of a first order differential equation

4.2.6    formulate differential equations, including the logistic equation that will arise in, for example, chemistry, biology and economics, in situations where rates are involved

**Modelling motion**

4.2.7    consider and solve problems involving motion in a straight line with both constant and non-constant

acceleration, including simple harmonic motion and the use of expressions , $\dfrac{dv}{dt}$, $v\dfrac{dv}{dx}$ and $\dfrac{d}{dx}\left(\dfrac{1}{2}v^2\right)$

for acceleration

## Topic 4.3: Statistical inference (15 hours)

**Sample means**

4.3.1    examine the concept of the sample mean $\overline{X}$ as a random variable whose value varies between samples where $X$ is a random variable with mean $\mu$ and the standard deviation $\sigma$

4.3.2    simulate repeated random sampling, from a variety of distributions and a range of sample sizes, to illustrate properties of the distribution of $\overline{X}$ across samples of a fixed size $n$, including its mean $\mu$ its

standard deviation $\dfrac{\sigma}{\sqrt{n}}$ (where $\mu$ and $\sigma$ are the mean and standard deviation of $X$), and its

approximate normality if $n$ is large

4.3.3    simulate repeated random sampling, from a variety of distributions and a range of sample sizes, to

illustrate the approximate standard normality of $\dfrac{\overline{X} - \mu}{s/\sqrt{n}}$ for large samples $(n \geq 30)$, where $s$ is the

sample standard deviation

**Confidence intervals for means**

4.3.4   examine the concept of an interval estimate for a parameter associated with a random variable

4.3.5   examine the approximate confidence interval $\left( \overline{X} - \dfrac{zs}{\sqrt{n}}, \overline{X} + \dfrac{zs}{\sqrt{n}} \right)$ as an interval estimate for the

population mean $\mu$, where $z$ is the appropriate quantile for the standard normal distribution

4.3.6   use simulation to illustrate variations in confidence intervals between samples and to show that most but not all confidence intervals contain $\mu$

4.3.7   use $\overline{x}$ and $s$ to estimate $\mu$ and $\sigma$ to obtain approximate intervals covering desired proportions of values of a normal random variable, and compare with an approximate confidence interval for $\mu$

# School-based assessment

The *Western Australian Certificate of Education (WACE) Manual* contains essential information on principles, policies and procedures for school-based assessment that needs to be read in conjunction with this syllabus.

Teachers design school-based assessment tasks to meet the needs of students. The table below provides details of the assessment types for the Mathematics Specialist ATAR Year 12 syllabus and the weighting for each assessment type.

**Assessment table – Year 12**

| Type of assessment | Weighting |
|---|---|
| **Response**<br>Students respond using knowledge of mathematical facts, concepts and terminology, applying problem solving skills and algorithms. Tasks can include: tests, assignments, quizzes and observation checklists. Tests are administered under controlled and timed conditions. | 40% |
| **Investigation**<br>Students use the mathematical thinking process to plan, research, conduct and communicate the findings of an investigation. They can investigate problems to identify the underlying mathematics, or select, adapt and apply models and procedures to solve problems. This assessment type provides for the assessment of the mathematical thinking process using course-related knowledge and modelling skills.<br>Evidence can include: observation and interview, written work or multimedia presentations. | 20% |
| **Examination**<br>Students apply mathematical understanding and skills to analyse, interpret and respond to questions and situations. Examinations provide for the assessment of conceptual understandings, knowledge of mathematical facts and terminology, problem-solving skills, and the use of algorithms.<br>Examination questions can range from those of a routine nature, assessing lower level concepts, through to those that require responses at the highest level of conceptual thinking. Students may be asked questions for which they may need to construct proofs and make conjectures.<br>Typically conducted at the end of each semester and/or unit and reflecting the examination design brief for this syllabus. | 40% |

Teachers are required to use the assessment table to develop an assessment outline for the pair of units.

The assessment outline must:

- include a set of assessment tasks

- include a general description of each task

- indicate the unit content to be assessed

- indicate a weighting for each task and each assessment type

- include the approximate timing of each task (for example, the week the task is conducted, or the issue and submission dates for an extended task).

In the assessment outline for the pair of units:

- each assessment type must be included at least twice

- the response type must include a minimum of two tests.

The set of assessment tasks must provide a representative sampling of the content for Unit 3 and Unit 4.

Assessment tasks not administered under test/controlled conditions require appropriate validation/authentication processes. This may include observation, annotated notes, checklists, interview, presentations or in-class tasks assessing related content and processes.

## Grading

Schools report student achievement in terms of the following grades:

| Grade | Interpretation |
|-------|----------------|
| A | Excellent achievement |
| B | High achievement |
| C | Satisfactory achievement |
| D | Limited achievement |
| E | Very low achievement |

The teacher prepares a ranked list and assigns the student a grade for the pair of units. The grade is based on the student's overall performance as judged by reference to a set of pre-determined standards. These standards are defined by grade descriptions and annotated work samples. The grade descriptions for the Mathematics Specialist ATAR Year 12 syllabus are provided in Appendix 1. They can also be accessed, together with annotated work samples, through the Guide to Grades link on the course page of the Authority website at www.scsa.wa.edu.au.

To be assigned a grade, a student must have had the opportunity to complete the education program, including the assessment program (unless the school accepts that there are exceptional and justifiable circumstances).

Refer to the *WACE Manual* for further information about the use of a ranked list in the process of assigning grades.

# ATAR course examination

All students enrolled in the ATAR Mathematics Specialist Year 12 course are required to sit the ATAR course examination. The examination is based on a representative sampling of the content for Unit 3 and Unit 4.

Details of the ATAR course examination are prescribed in the examination design brief on the following page.

Refer to the WACE Manual for further information.

## Examination design brief – Year 12

This examination consists of two sections.

**Section One: calculator-free**

**Time allowed**
Reading time before commencing work:     five minutes
Working time for section:                        fifty minutes

**Permissible items**
Standard items:   pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters
Special items:     nil

**Additional information**
Changeover period during which the candidate is not permitted to work: up to 15 minutes

**Section Two: calculator-assumed**

**Time allowed**
Reading time before commencing work:     ten minutes
Working time for section:                        one hundred minutes

**Permissible items**
Standard items:   pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters
Special items:     drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators, which can include scientific, graphic and Computer Algebra System (CAS) calculators, are permitted in this ATAR course examination

**Provided by the supervisor**
A formula sheet

**Additional information**
It is assumed that candidates sitting this examination have a calculator with CAS capabilities for Section Two.
The examination assesses the syllabus content areas using the following percentage ranges. These apply to the whole examination rather than individual sections.

| Content area | Percentage of exam |
| --- | --- |
| Complex numbers | 15–20% |
| Functions and graphs | 10–15% |
| Vectors in three dimensions | 15–20% |
| Integration and application of integration | 15–20% |
| Rates of change and differential equations | 15–20% |
| Statistical inference | 10–15% |

The candidate is required to demonstrate knowledge of mathematical facts, conceptual understandings, use of algorithms, use and knowledge of notation and terminology, and problem-solving skills.

Questions can require the candidate to solve problems, investigate mathematical patterns, make and test conjectures, and generalise and prove mathematical relationships. Questions can require the candidate to apply concepts and relationships to unfamiliar problem-solving situations, choose and use mathematical models with adaptations, compare solutions, and present conclusions. Examination questions can range from those of a routine nature, assessing lower level concepts, through to those that require responses at the highest level of conceptual thinking.

Instructions to candidates indicate that, for any question or part question worth more than two marks, valid working or justification is required to receive full marks.

| Section | Supporting information |
|---|---|
| **Section One: calculator-free**<br>35% of the total examination<br>5–10 questions<br>Suggested working time: 50 minutes | Questions examine content and procedures that can reasonably be expected to be completed without the use of a calculator i.e. without undue emphasis on algebraic manipulations or time consuming calculations.<br><br>The candidate is required to provide answers that include: calculations, tables, graphs, interpretation of data, descriptions and/or conclusions.<br><br>Stimulus material can include: diagrams, tables, graphs, drawings, print text and/or data gathered from the media. |
| **Section Two: calculator-assumed**<br>65% of the total examination<br>8–13 questions<br>Suggested working time: 100 minutes | Questions examine content and procedures for which the use of a calculator is assumed.<br><br>The candidate can be required to provide answers that include: calculations, tables, graphs, interpretation of data, descriptions and/or conclusions.<br><br>Stimulus material can include: diagrams, tables, graphs, drawings, print text and/or data gathered from the media.<br><br>The candidate can be required to investigate theoretical situations involving mathematical concepts and relationships, for which they need to generalise, construct proofs and make conjectures.<br><br>The candidate can be required to solve problems from unfamiliar situations, choosing and using mathematical models with adaptations where necessary, comparing their solutions with the situations concerned, and then presenting their findings in context. |

# Appendix 1 – Grade descriptions Year 12

<table>
<tr>
<td rowspan="5" align="center"><strong>A</strong></td>
<td>
<strong>Identifies and organises relevant information</strong><br>
Identifies and organises information from previous parts of a problem to carry through and solve new and different problems.<br>
Defines variables and equations from text and diagrams.<br>
Organises data in a concise, clear format and appropriately presents it in tabular, diagrammatic and/or graphical form.<br>
Identifies the underlying assumptions related to the relevant mathematics of an investigation.
</td>
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<tr>
<td>
<strong>Chooses effective models and methods and carries through the methods correctly</strong><br>
Selects an appropriate strategy and applies mathematical knowledge to solve non-routine problems.<br>
Generalises and extends models from previous parts of the question.<br>
Translates between representations in unpractised ways.<br>
Selects appropriate calculator techniques to solve multi-step problems in unfamiliar contexts.<br>
Solves unstructured problems and carries through an extended response, using deductive reasoning.<br>
Produces results, carries out analysis and generalises in situations requiring investigative techniques.
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<tr>
<td>
<strong>Follows mathematical conventions and attends to accuracy</strong><br>
Consistently uses mathematical conventions to link expressions in clearly defined steps that are easily followed.<br>
Works fluently with exact values such as surds, radian values or natural logarithms and exponentials, and expresses answers accurately using these forms as appropriate.<br>
Completes concise and accurate solutions to mathematical problems set in applied and theoretical contexts.
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<tr>
<td>
<strong>Links mathematical results to data and contexts to reach reasonable conclusions</strong><br>
Identifies and explains the limitations of complex models.<br>
Recognises the domain as implied by the context of the question, and excludes any results outside it.<br>
Interprets the result and draws the correct conclusion about the effect of changing conditions.<br>
Uses counter-examples and general cases in mathematical analysis of an investigation.
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<tr>
<td>
<strong>Communicates mathematical reasoning, results and conclusions</strong><br>
Sets out the steps of the solution using deductive reasoning in a clear and logical sequence, including suitable justification and explanation of methods and processes used.<br>
Adds a detailed diagram to illustrate and use in the solution of a problem.<br>
Presents work with the final answer clearly identified, using the correct units and related to the context of the question.<br>
Communicates investigation findings with a comprehensive interpretation of mathematical results in the context of the investigation.
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</tr>
</table>

| | | |
|---|---|---|
| **B** | **Identifies and organises relevant information** |
| | Identifies and organises key information from previous parts of a problem and brings it together to solve subsequent parts of the problem. |
| | Draws a diagram and labels it with appropriate variables. |
| | Organises data clearly and appropriately presents it in tabular, diagrammatic and/or graphical form. |
| | Identifies suitable variables and constant parameters related to various aspects of an investigation. |

**Chooses effective models and methods and carries the methods through correctly**

Selects an appropriate strategy and applies mathematical knowledge to solve simple non-routine problems.

Translates between representations in practised ways.

Selects appropriate calculator techniques to solve multi-step problems.

Attempts to analyse and calculate specific cases of generalisation in situations requiring investigative techniques.

**Follows mathematical conventions and attends to accuracy**

Interprets and uses mathematical terminology, symbols and conventions in unpractised situations.

Defines introduced variables.

Uses exact values, such as surds and radian values to express answers accurately when specified.

Completes mostly accurate solutions to mathematical problems set in applied and theoretical contexts.

**Links mathematical results to data and contexts to reach reasonable conclusions**

Identifies and explains the limitations of simple models.

Takes account of the domain as defined in the problem, and excludes results outside it.

Links the effect of changing conditions to the original solution.

Uses examples in mathematical analysis of an investigation and draws valid conclusions related to a given context.

**Communicates mathematical reasoning, results and conclusions**

Carries through calculations and simplifications in a clear sequence, showing a logical line of reasoning.

Defines variables associated with a given diagram and uses these in the working of a problem.

Presents work with the final answer clearly identified and using the correct units.

Communicates investigation findings in a systematic and concise way using mathematical language and relating the solution to the original problem or statement.

| C | **Identifies and organises relevant information from information that is relatively narrow in scope**<br>Identifies and organises key information needed to solve a familiar problem.<br>Identifies variables in a given diagram or draws a simple diagram from given information.<br>Organises some data and presents it in tabular, diagrammatic and/or graphical form.<br>Identifies some mathematical content related to various aspects of an investigation in a given context. |
|---|---|
| | **Chooses effective models and methods and carries through the methods correctly**<br>Selects from a range of strategies and formulae and applies mathematical knowledge in practised ways to solve routine problems.<br>Recognises and uses information in different representations.<br>Uses familiar calculator applications to solve routine problems.<br>Selects an appropriate strategy to carry out analysis in situations requiring investigative techniques. |
| | **Follows mathematical conventions and attends to accuracy**<br>Recognises and applies mathematical definitions, rules and procedures in practised situations.<br>Applies basic conventions for diagrams and graphs.<br>Uses calculus, vector and complex number notation correctly.<br>Rounds to suit context and specified accuracy.<br>Generates some accurate and generally complete solutions to mathematical problems set in applied and theoretical contexts. |
| | **Links mathematical results to data and contexts to reach reasonable conclusions**<br>Identifies and describes limitations of simple models.<br>Shows some recognition of the domain as defined in the problem.<br>Recognises that changing conditions will affect the outcome.<br>Makes inferences from analysis and uses these to draw conclusions related to a given context for investigation. |
| | **Communicates mathematical reasoning, results and conclusions**<br>Shows adequate working and supports answers with simple or routine statements.<br>Relates the working to a labelled diagram that has been given as part of the question.<br>Presents work that includes the required answer but does not clearly identify it as such.<br>Communicates investigation findings in a systematic way using some mathematical expression and everyday language. |

| | |
|---|---|
| **D** | **Identifies and organises relevant information**<br>Identifies and attempts to use given information to solve a simple routine problem.<br>Identifies variables in a simple diagram.<br>Displays data using an inappropriate presentation format.<br>Identifies limited mathematical content of an investigation.<br><br>**Chooses effective models and methods and carries the methods through correctly**<br>Follows an appropriate strategy to solve practised problems.<br>Deals with information in familiar representations only.<br>Uses a calculator for straightforward problems.<br>Attempts to select an appropriate strategy to carry out analysis in situations requiring investigative techniques.<br><br>**Follows mathematical conventions and attends to accuracy**<br>Applies limited mathematical conventions to practised problems.<br>Expresses answers with limited accuracy.<br>Generates partly accurate and generally incomplete solutions to mathematical problems set in applied and theoretical contexts.<br><br>**Links mathematical results to data and contexts to reach reasonable conclusions**<br>Identifies limitations of simple models and, on occasions, recognises specified conditions.<br>Makes some inferences from analysis of an investigation.<br><br>**Communicates mathematical reasoning, results and conclusions**<br>Shows some working in an attempt to answer simple questions.<br>Sets out calculations in a manner that is difficult to check for accuracy.<br>Presents working with no clear indication of the final answer evident.<br>Offers simple conclusions that are not supported by data or calculations. |
| **E** | Does not meet the requirements of a D grade and/or has completed insufficient assessment tasks to be assigned a higher grade. |

# Appendix 2 – Glossary

This glossary is provided to enable a common understanding of the key terms in this syllabus.

**Unit 3**

### Complex numbers

### Argument (abbreviated arg)
If a complex number $z$ is represented by a point P in the complex plane then the argument of $z$,
denoted $\arg z$, is the angle $\theta$ that OP makes with the positive real axis $O_x$,
with the angle measured anticlockwise from $O_x$.
The principal value of the argument is the one in the interval $(-\pi, \pi]$.

### Complex arithmetic
If $z_1 = x_1 + y_1 i$ and $z_2 = x_2 + y_2 i$
$$\Rightarrow z_1 + z_2 = (x_1 + x_2) + (y_1 + y_2)i$$
$$\Rightarrow z_1 - z_2 = (x_1 - x_2) + (y_1 - y_2)i$$
$$\Rightarrow z_1 \times z_2 = (x_1 x_2 - y_1 y_2) + (x_1 y_2 + x_2 y_1)i$$
$z_1 \times (0 + 0i) = 0$  *Note* : $0 + 0i$ is usually written as 0
$z_1 \times (1 + 0i) = z_1$ *Note* : $1 + 0i$ is usually written as 1

### Complex conjugate
For any complex number $z = x + iy$, its conjugate is $\overline{z} = x - iy$. the following properties hold
$$\overline{(z_1 z_2)} = \overline{z}_1 \times \overline{z}_2$$
$$\overline{\left(\frac{z_1}{z_2}\right)} = \frac{\overline{z}_1}{\overline{z}_2}$$
$$z \times \overline{z} = \left|z^2\right|$$
$z + \overline{z}$ is real.

### De Moivre's theorem

For all integers $n$, $(\cos \theta + i\sin \theta)^n = \cos n\theta + i\sin n\theta$.

### Modulus (absolute value) of a complex number
If $z$ is a complex number and $z = x + iy$,
then the modulus of $z$ is the distance of $z$ from the origin in the Argand plane.
The modulus of $z$ denoted by $|z| = \sqrt{x^2 + y^2}$.

### Polar form of a complex number
For a complex number $z$, let $\theta = \arg z$, then $z = r(\cos \theta + i\sin \theta)$ is the polar form of $z$.

**Root of unity (nth root of unity)**

For a complex number $z$ such that $z^n = 1$ the $n^{th}$ roots of unity are:

$z_k = \cos\left(\dfrac{2k\pi}{n}\right) + i\sin\left(\dfrac{2k\pi}{n}\right)$ where $k = 0, 1, 2, \ldots, n-1$.

The points in the complex plane representing roots of unity lie on the unit circle.

The cube roots of unity are:

$z_0 = \cos(0) + i\sin(0) \qquad = 1,$

$z_1 = \cos\left(\dfrac{2\pi}{3}\right) + i\sin\left(\dfrac{2\pi}{3}\right) \quad = -\dfrac{1}{2} + \dfrac{\sqrt{3}}{2}i$

$z_2 = \cos\left(\dfrac{4\pi}{3}\right) + i\sin\left(\dfrac{4\pi}{3}\right) \quad = -\dfrac{1}{2} - \dfrac{\sqrt{3}}{2}i.$

## Functions and sketching graphs

### Rational function

A rational function is a function such that $f(x) = \dfrac{g(x)}{h(x)}$ where $g(x)$ and $h(x)$ are polynomials. Usually

$g(x)$ and $h(x)$ are chosen so as to have no common factor of degree greater than or equal to 1, and the domain of $f$ is usually taken to be $R\backslash\{x: h(x) \neq 0\}$

## Vectors in three dimensions

### Addition of vectors

In component form if $\mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$ and $\mathbf{b} = b_1\mathbf{i} + b_2\mathbf{j} + b_3\mathbf{k}$ then

$\mathbf{a} + \mathbf{b} = (a_1 + b_1)\mathbf{i} + (a_2 + b_2)\mathbf{j} + (a_3 + b_3)\mathbf{k}.$

### Vector equation of a straight line

Let $\mathbf{a}$ be the position vector of a point on a line $l$, and $\mathbf{u}$ any vector with direction parallel to the line.

The line consists of all points P whose position vector $\mathbf{p}$ is given by;

$\mathbf{p} = \mathbf{a} + t\mathbf{u}$ for some real number $t$.

Also given two points A and B on the line with the position vectors $\mathbf{a}$ and $\mathbf{b}$ respectively

the equation of the line passing through these points can be written as;

$\mathbf{p} = \mathbf{a} + t(\mathbf{b} - \mathbf{a})$ for some real number $t$.

### Vector equation of a plane

Let $\mathbf{a}$ be a position vector of a point $A$ in the plane, and $\mathbf{n}$ a normal vector to the plane. Then the plane consists of all points $P$ whose position vector $\mathbf{p}$ $(\boldsymbol{p} - \boldsymbol{a}) \bullet \boldsymbol{n} = 0.$ satisfies

This equation may also be written as $\boldsymbol{p} \bullet \boldsymbol{n} = \boldsymbol{a} \bullet \boldsymbol{n}$, a constant.

(If the normal vector $\mathbf{n}$ is the vector (l, m, n) in ordered triple notation and the scalar product $\boldsymbol{a} \bullet \boldsymbol{n} = k$ this gives the Cartesian equation $lx + my + nz = k$ for the plane.)

### Vector function

In this course, a vector function is one that depends on a single real number parameter $t$, often representing time, producing a vector $\mathbf{r}$ ($t$) as the result. In terms of the standard unit vectors $\mathbf{i}, \mathbf{j}, \mathbf{k}$ of three- dimensional space, the vector-valued functions of this specific type are given by expressions such as $\mathbf{r}(t) = f(t)\,\mathbf{i} + g(t)\mathbf{j} + h(t)\mathbf{k}$ where $f$, $g$ and $h$ are real valued functions giving coordinates.

**Scalar product**

If $\mathbf{a} = (a_1, a_2, a_3)$ and $\mathbf{b} = (b_1, b_2, b_3)$ then the scalar product $\mathbf{a} \bullet \mathbf{b}$ is the real number where;

$\mathbf{a} \bullet \mathbf{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$.

When expressed in **i, j, k** notation, $\mathbf{a} = (a_1 \mathbf{i}, a_2 \mathbf{j}, a_3 \mathbf{k})$ and $\mathbf{b} = (b_1 \mathbf{i}, b_2 \mathbf{j}, b_3 \mathbf{k})$,

then $\mathbf{a} \bullet \mathbf{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$.

**Vector product (cross product)**

When expressed in $\mathbf{i}, \mathbf{j}, \mathbf{k}$ notation, $\mathbf{a} = a_1 \mathbf{i} + a_2 \mathbf{j} + a_3 \mathbf{k}$ and $\mathbf{b} = b_1 \mathbf{i} + b_2 \mathbf{j} + b_3 \mathbf{k}$, then;

$\mathbf{a} \times \mathbf{b} = (a_2 b_3 - a_3 b_2) \mathbf{i} + (a_3 b_1 - a_1 b_3) \mathbf{j} + (a_1 b_2 - a_2 b_1) \mathbf{k}$

The vector cross product has the following geometric interpretation.

Let $\mathbf{a}$ and $\mathbf{b}$ be two non-parallel vectors, then $|\mathbf{a} \times \mathbf{b}|$ is the area of the parallelogram defined by $\mathbf{a}$ and $\mathbf{b}$ and where $\mathbf{a} \times \mathbf{b}$ is a vector normal to this parallelogram.

(The cross product of two parallel vectors is the zero vector.)

**Unit 4**

## Rates of change and differential equations

### Implicit differentiation

When variables $x$ and $y$ satisfy a single equation, this may define $y$ as a function of $x$ even though there is no explicit formula for $y$ in terms of $x$. Implicit differentiation consists of differentiating each term of the equation as it stands and making use of the chain rule. This can lead to a formula for $\dfrac{dy}{dx}$

For example:

$$x^2 + xy^3 - 2x + 3y = 0, \text{ then } \frac{d}{dx}\left(x^2 + xy^3 - 2x + 3y = 0\right)$$

$$\Rightarrow 2x + y^3 + x.3y^2\frac{dy}{dx} - 2 + 3\frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx} = \frac{2 - 2x - y^3}{3xy^2 + 3}.$$

### Logistic equation

The logistic equation has applications in a range of fields, including biology, biomathematics, economics, chemistry, mathematical psychology, probability, and statistics.

One form of this differential equation is:

$$\frac{dy}{dt} = ay - by^2 \text{ (where } a > 0 \text{ and } b > 0)$$

The general solution of this is:

$$y = \frac{a}{b + ce^{-at}} \text{ where } c \text{ is an arbitrary constant.}$$

### Separation of variables

Differential equations of the form $\dfrac{dy}{dx} = g(x)h(y)$ can be rearranged as long as $h(y) \neq 0$

to obtain $\dfrac{1}{h(y)}\dfrac{dy}{dx} = g(x)$

### Slope field

Slope field (direction or gradient field) is a graphical representation of the solutions of a linear first-order differential equation in which the derivative at a given point is represented by a line segment with the corresponding slope.

## Statistical inference

### Continuous random variable

A random variable $X$ is called continuous if its set of possible values consists of intervals, and the chance that it takes any point value is zero. (In symbols, if $P(X = x) = 0$ for every real number $x$).

A random variable is continuous if, and only if, its cumulative probability distribution function can be expressed as an integral of a function.

### Probability density function

The probability of a continuous random variable $X$ being located in a given interval $[a, b]$ is denoted

by $Pr(a \leq X \leq b) = \int_b^a f(x)dx$ where $f(x)$ is defined as the probability density function $(\text{pdf})$.

The probability density function is therefore the derivative of the cumulative probability function.

### Precision

Precision is a measure of how close an estimator is expected to be to the true value of the parameter it purports to estimate.

### Independent and identically distributed observations

For independent observations, the value of any one observation has no effect on the chance of values for all the other observations. For identically distributed observations, the chances of the possible values of each observation are governed by the same probability distribution.

### Random sample

A random sample is a set of data in which the value of each observation is governed by some chance mechanism that depends on the situation. The most common situation in which the term "random sample" is used refers to a set of independent and identically distributed observations.

The sample mean is the arithmetic average of the sample values.